

# SISMOGRAMMES HORS-CURRICULUM

*Des Tsunamis aux Ondes de Gravité Aériennes*

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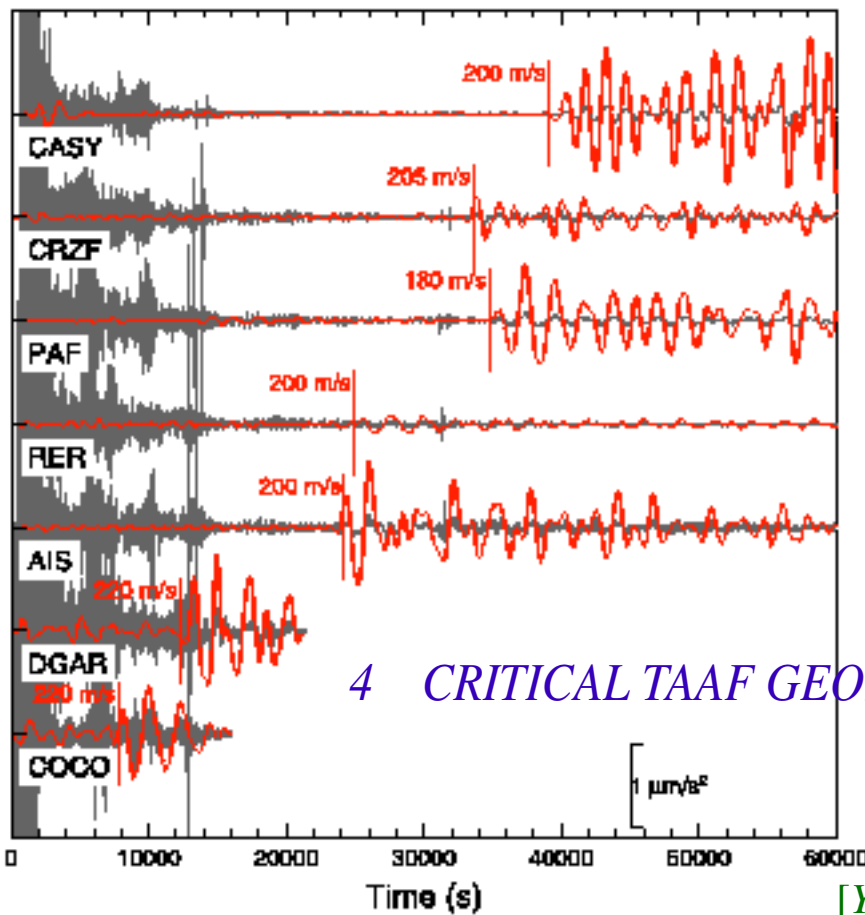
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Colloque "40 ans de GEOSCOPE"  
Institut de Physique du Globe  
Paris, Jeudi 30 juin 2022

# TSUNAMI RECORDED ON SEISMOMETERS

- Horizontal long-period seismometers (GEOSCOPE, IRIS...) record ultra-long period oscillations following arrival of 2004 tsunami at nearby shores [R. Kind, 2005].
- Energy is mostly between 800 and 3000 seconds
- Amplitude of equivalent displacement is **centimetric**

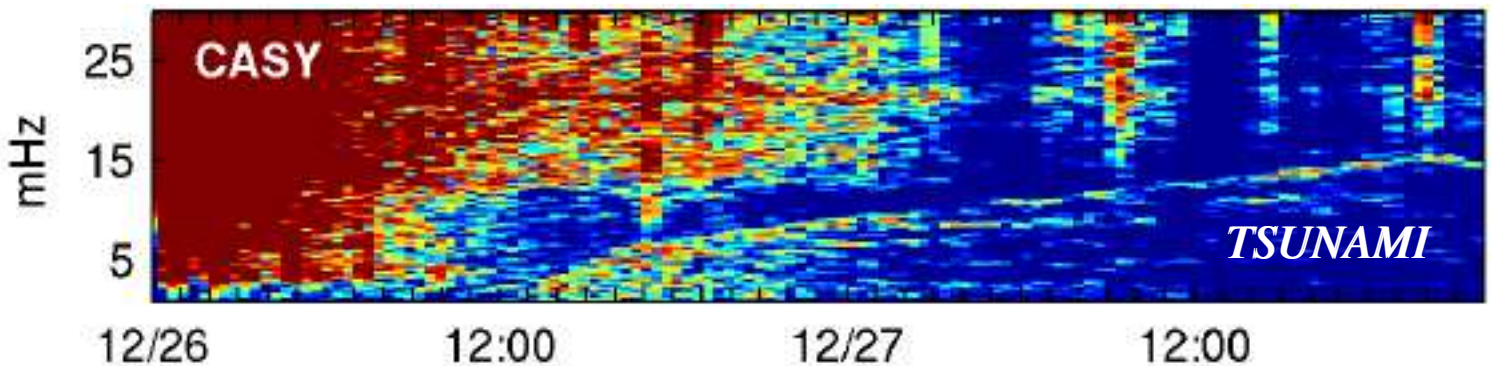
Crozet  
Kerguelen  
Réunion  
Amsterdam



4 CRITICAL TAAF GEOSCOPE STATIONS



[Yuan et al., 2005]



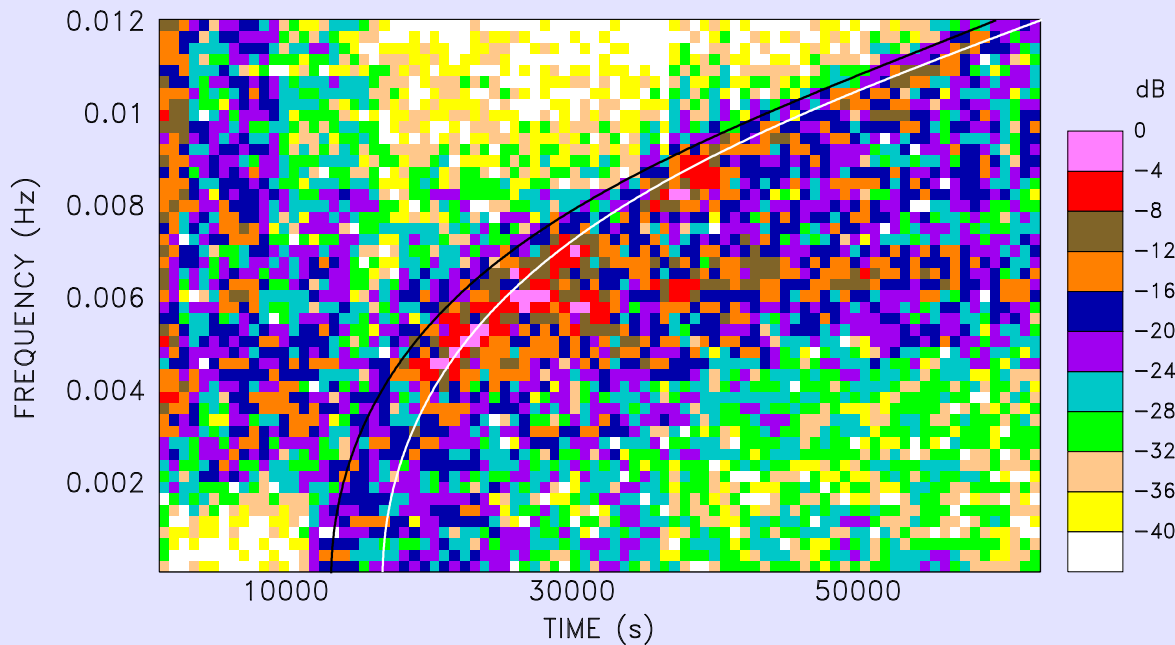
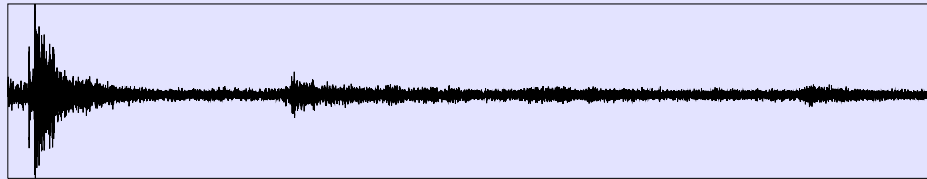
[Hanson and Bowman, 2005]

# Ile Amsterdam, 26 Dec. 2004

## RAW N-S GEOSCOPE RECORD (VH Channel)

AINN 04 361 0 2 15.1020

Peak-to-peak = 0.233E+06 du



**NOTE STRONG HIGH-FREQUENCY TSUNAMI COMPONENTS**

Dispersed energy resolved down to  $T = 80$  s.

**CAN WE QUANTIFY SUCH RECORDS ?**



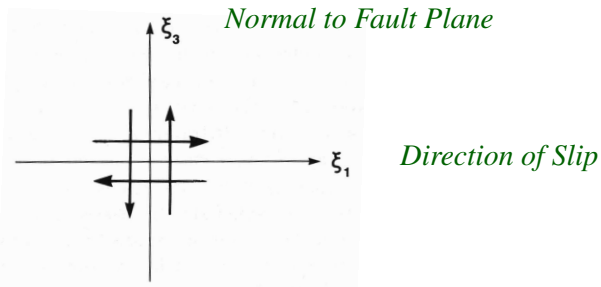
## EXCITATION OF TSUNAMI in NORMAL MODE FORMALISM

- *Gilbert* [1970] has shown that the response of the Earth to a point source consisting of a single force  $\mathbf{f}$  can be expressed as a summation over all of its normal modes

$$\mathbf{u}(r, t) = \sum_N \mathbf{s}_n(\mathbf{r}) \left( \mathbf{s}_n^*(\mathbf{r}_s) \cdot \mathbf{f}(\mathbf{r}_s) \right) \cdot \frac{1 - \cos \omega_n t \exp(-\omega_n t / 2Q_n)}{\omega_n^2},$$

the *EXCITATION* of each mode being proportional to the *scalar product of the force  $\mathbf{f}$  by the eigen-displacement  $\mathbf{s}$  at location  $\mathbf{r}_s$* .

- Now, an *EARTHQUAKE* is represented by a system of forces called a *double-couple*:



The response of the Earth to an earthquake is thus

$$\mathbf{u}(r, t) = \sum_N \mathbf{s}_n(\mathbf{r}) \left( \boldsymbol{\varepsilon}_n^*(\mathbf{r}_s) : \mathbf{M}(\mathbf{r}_s) \right) \cdot \frac{1 - \cos \omega_n t \exp(-\omega_n t / 2Q_n)}{\omega_n^2}$$

where the *EXCITATION* is the *scalar product* of the earthquake's **MOMENT  $\mathbf{M}$**  with the local *eigenstrain  $\boldsymbol{\varepsilon}$*  at the source  $\mathbf{r}_s$ .

This formula is directly applicable to the case of a tsunami represented by normal modes of the Earth.



## QUANTIFYING the SEISMIC RECORD at CASY

Assume that seismic record (e.g., at CASY) reflects response of seismometer to the *deformation of the ocean bottom*.

**FORGET THE ISLAND (or continent)!**

- Use *Gilbert's* [1980] combination of displacement, tilt and gravity;

Apparent Horizontal Acceleration (*Gilbert's* [1980] Notation):

$$AV = \omega^2 V - r^{-1} L (gU + \Phi)$$

or (*Saito's* [1967] notation):

$$y_3^{APP} = y_3 - \frac{1}{r \omega^2} \cdot (g y_1 - y_5)$$

- Use *Ward's* [1980] normal mode formalism;

Evaluate *Gilbert* response on solid side of ocean floor, and derive equivalent spectral amplitude of surface displacement  $y_1(\omega) = \eta(\omega)$ .

- Use *Okal and Titov's* [2005] Tsunami Magnitude, inspired from *Okal and Talandier's* [1989]  $M_m$ ;

- Apply to CASY record at maximum spectral energy ( $S(\omega) = 4000 \text{ cm}^2 \text{ s}$  at  $T = 800 \text{ s}$ ).

→ Find  **$M_0 = 1.7 \times 10^{30} \text{ dyn-cm}$** .

Published:  $1.15 \times 10^{30} \text{ dyn}^2 \text{ cm}$  [*Stein and Okal, 2005; Tsai et al., 2005*]

Acceptable, given the extreme nature of the approximations.

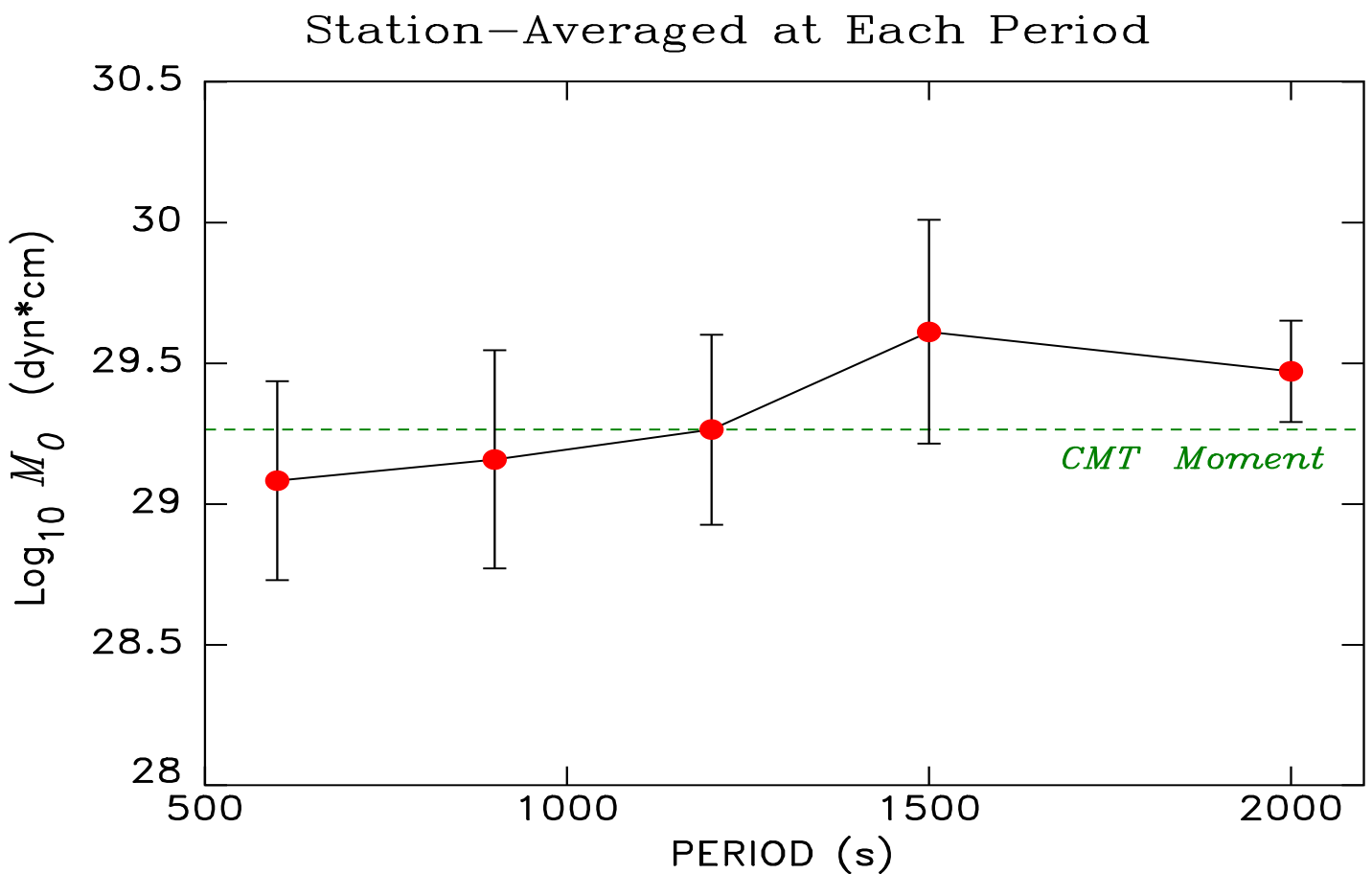
→ Suggests that the signal is just the expression of the horizontal deformation of the ocean floor, and that

**CASY functions in a sense like an OBS !!**

# MAULE, Chile, 2010

## 8 Seismic Stations — 12 Components

→ *In the 500–2000 s period range, the results are generally in agreement with the CMT scalar moment.*



*This supports the finding [Okal et al., 2010] that the Maule earthquake is **not a slow event**.*

→ At higher frequencies (not shown), the results would depend on the response of the individual island structure.

# A REMARKABLE ANTECEDENT



*Kuriles, 07 SEP 1918*

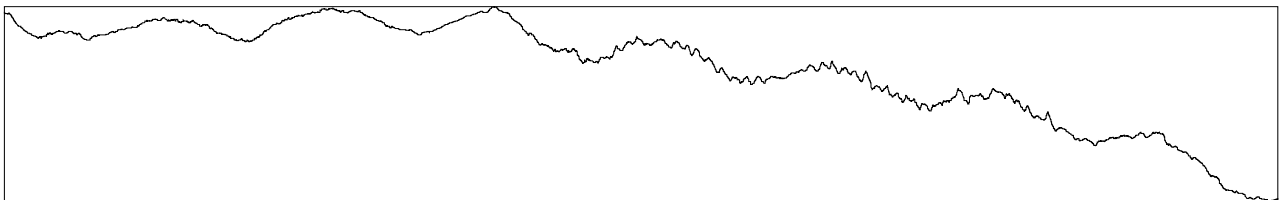
*at Apia [ex-German] Samoa*

as reported by *G. Angenheister [1920]*

Beim Vorüberziehen der Flutwellen des Kurilenbebens (etwa 9<sup>h</sup> nach Ankunft der seismischen Wellen in Apia) zeigte, der Wiechertsche Horizontal-Seismograph Neigungswellen von  $\frac{1}{2}$ <sup>h</sup>—1<sup>h</sup> Periode und bis zu 0".2 Amplitude.

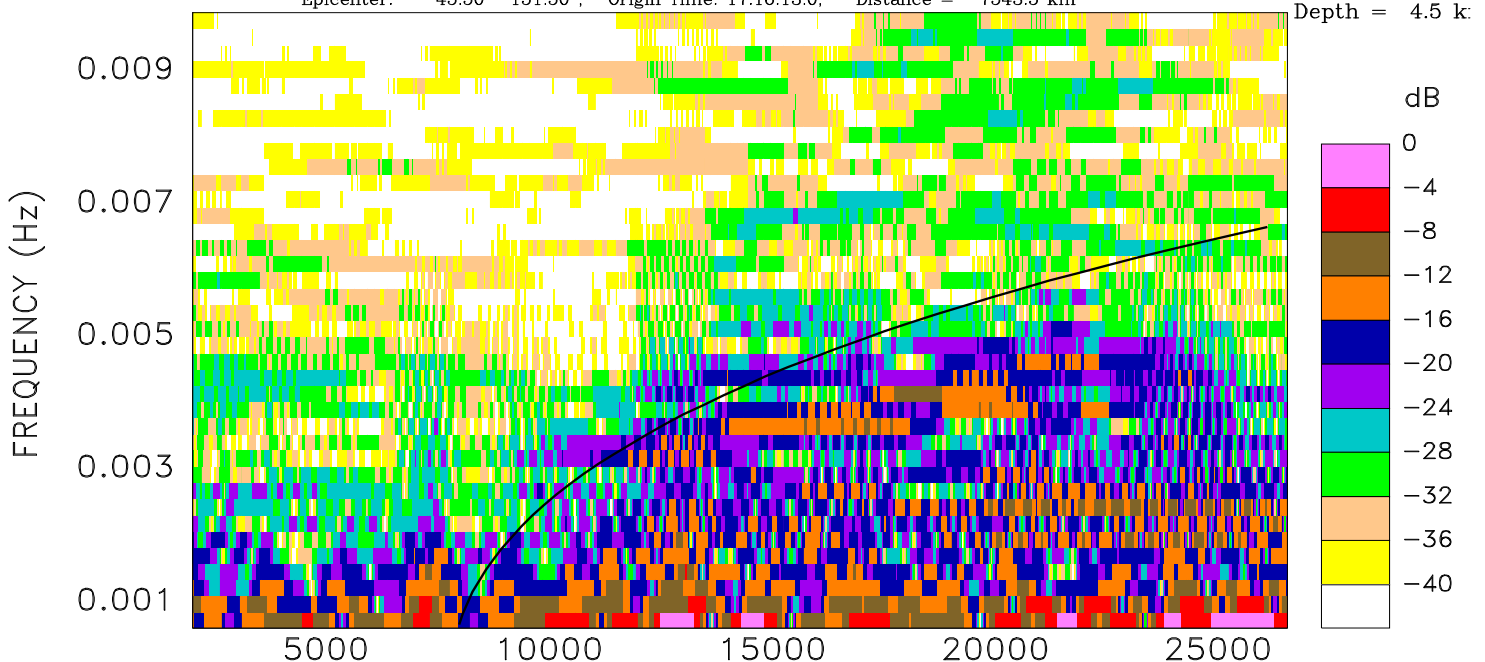
APIN 18 250 25 4 0.0000

Peak-to-peak = 0.607E+01 du



Epicenter: 45.50 151.50 ; Origin Time: 17:16:13.0; Distance = 7543.5 km

Depth = 4.5 k



Window = 4000.0 s; Slice = 15. s.  
F-min = 0.00050 Hz; F-max = 0.01000 Hz.  
[ Plot bounds: 0.61035E-03 0.98877E-02 ]

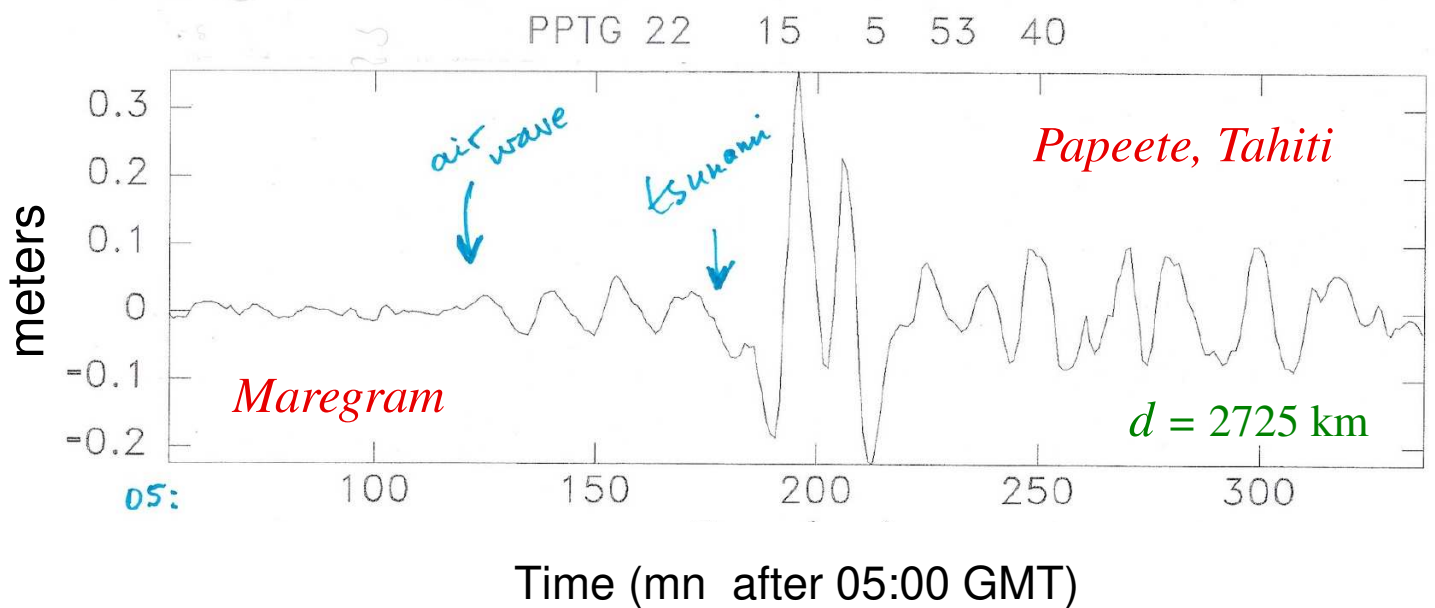
TIME (s)

Max. Spectral Amp. (0 dB) = 0.432E+03 du\*s  
Window used : 0.00 s; 28559.00 s.  
Plot window : 1992.50 s; 26547.50 s.

# 2022 HUNGA-TONGA VOLCANIC EXPLOSION

## *A very intriguing "tsunami"*

- At many locations of the Pacific, wave activity starts **BEFORE** the predicted arrival of the tsunami.

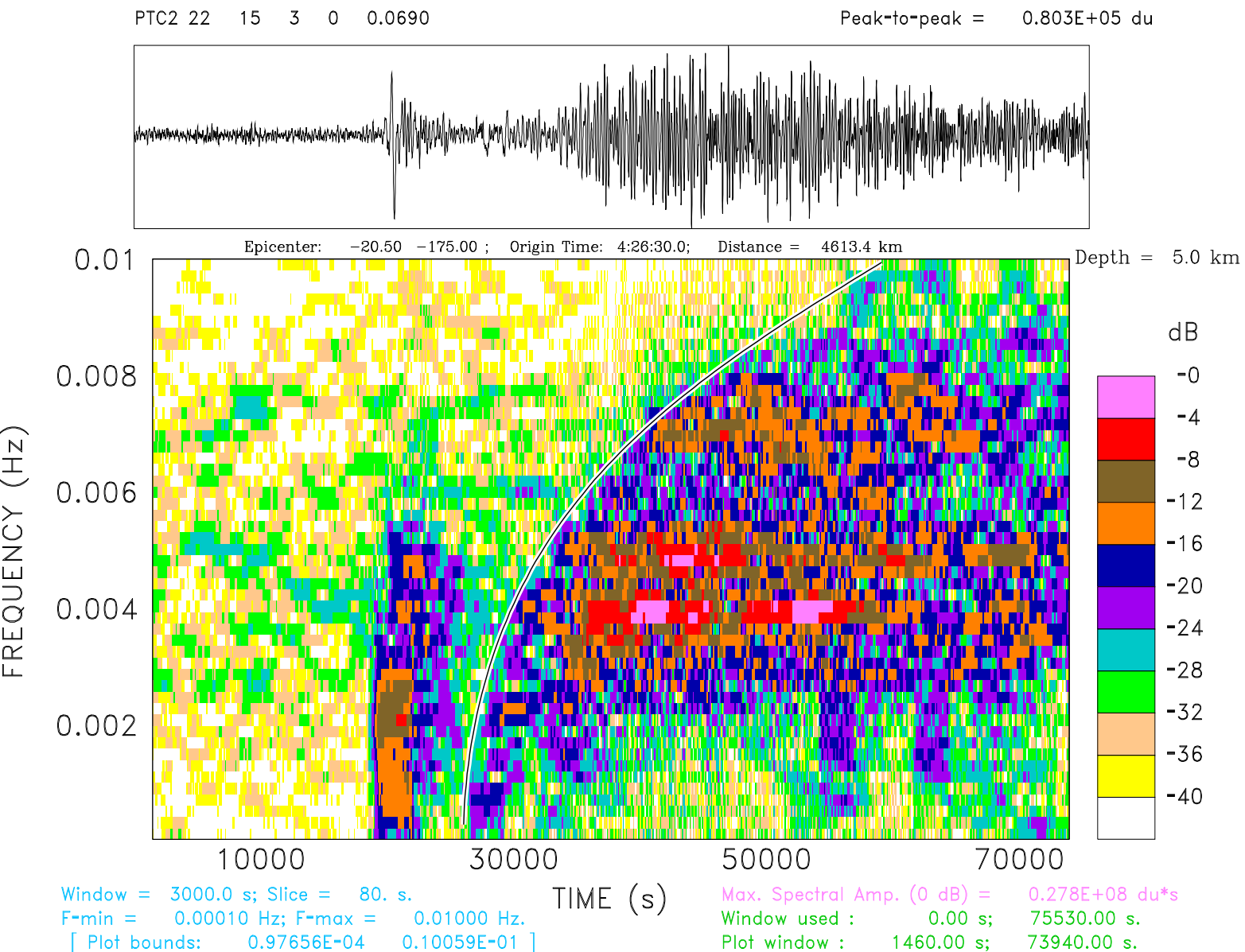


→ *This corresponds to an acoustic wave in the atmosphere, which is coupled to the water column, resulting in a disturbance of the sea surface.*

That wave, propagating at a typical velocity of **313 m/s**, is significantly precursory to the tsunami.

# A SPECTACULAR EXAMPLE OF THE TWO WAVES

is given by the EW seismic record at Pitcairn Island



## NOTE

- the perfect dispersion of the "true" tsunami, outside the Shallow-Water Approximation
- the weak dispersion of the air wave
- the strong spectral peak in the tail of the tsunami at ~4 mHz (stay tuned)
- the absence of conventional long-period seismic waves



~~A "new kid" on the block  
*among volcanic tsunami players ?*~~

*Perhaps RATHER*

**AN OLD REVENANT...**

# AIR-SEA WAVES OBSERVED DURING 1883 KRAKATAU EXPLOSION

TSUNAMI GENERATION by *Volcanic Explosions at Sea*

*Krakatau [Sunda Straits], 27 August 1883*

ANAK KRAKATAU, Sept. 2016



*Born 1927... and Still Growing !*



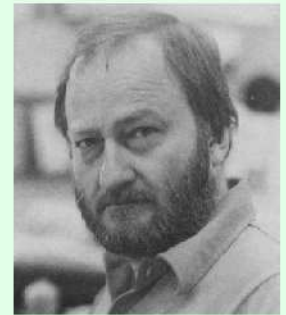
← 12 km →

A catastrophic tsunami killed 35,000 people in Batavia (Jakarta). *Nomambhoy and Satake [1995]* showed that it can be well modeled by an underwater explosion.

*The tsunami was reported recorded world-wide (on tidal gauges), which would seem to contradict the dispersive nature of the short wavelengths associated with sources of small dimensions...*



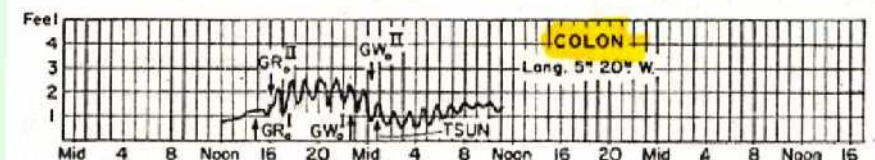
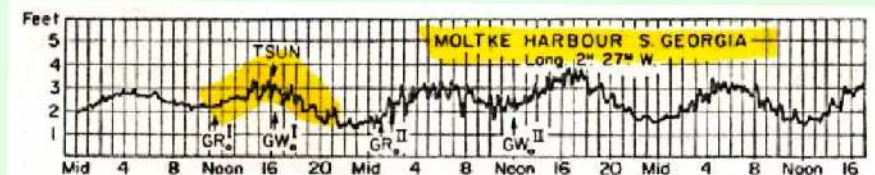
HOWEVER ...



*Press and Harkrider [1965, 1967]* had shown that the tsunami is actually triggered by an **air wave** generated by an atmospheric explosion, and re-exciting the ocean as it propagates.

This explains

- the propagation of the "tsunami" along great circle paths occasionally crossing... a continent!
- the occasional early arrival of the tsunami at distant tidal stations (**315 m/s as opposed to 200 m/s**).
- and allows an estimate of the power of the explosion (100 to 150 Mt).



# AIR-SEA WAVES OBSERVED DURING 1883 KRAKATAU EXPLOSION

TSUNAMI GENERATION by *Volcanic Explosions*

*Krakatau [Sunda Straits], 27 Aug 1883*

ANAK KRAKATAU, Sept. 20, 1883



*Born 1883*

**THIS PART of the ISLAND NO LONGER EXISTS since the 2018 ERUPTION!**

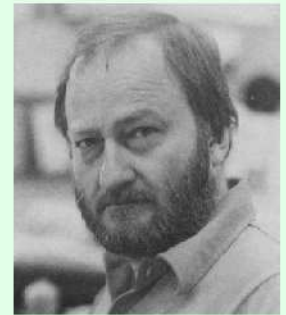
A catastrophe in 1883 (Jakarta). No one could be well modeled.

... people in Batavia [1995] showed that it can be modeled as a water explosion.

*The tsunami was reported recorded world-wide (on tidal gauges), which would seem to contradict the dispersive nature of the short wavelengths associated with sources of small dimensions...*



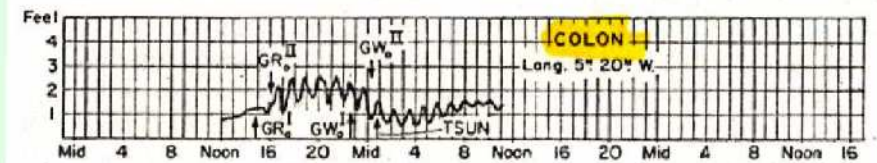
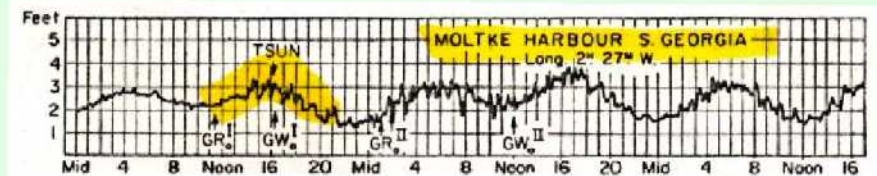
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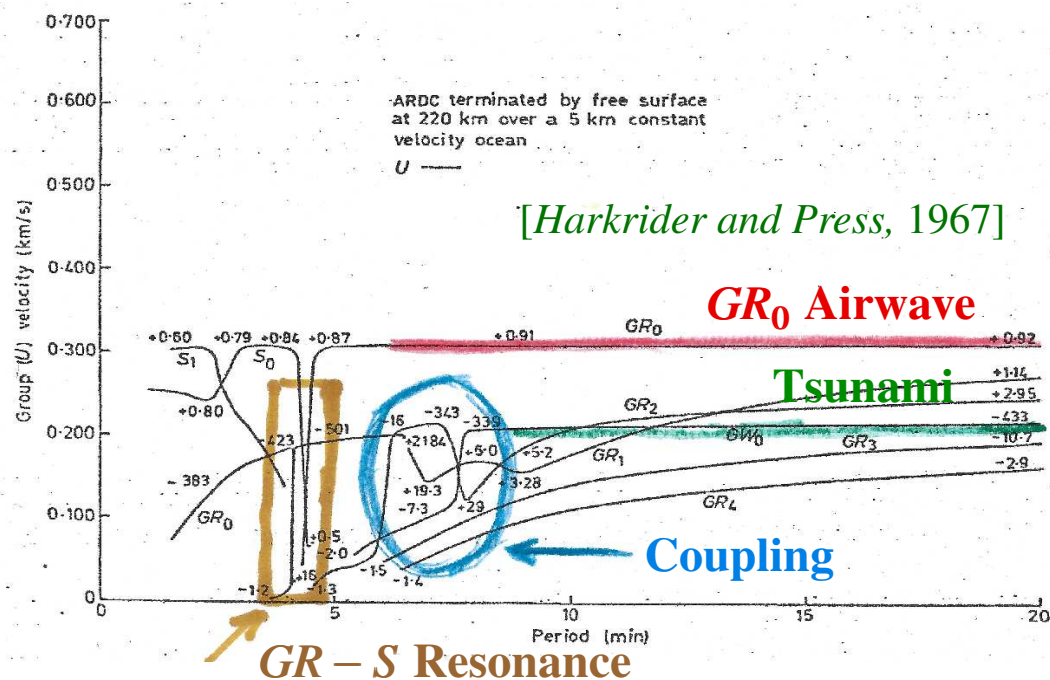


When the atmosphere is underlain by an ocean, the two families of modes feature different boundary conditions

- with their eigenfunctions penetrating the other medium.

→ But their dispersion remains [largely] unaffected.

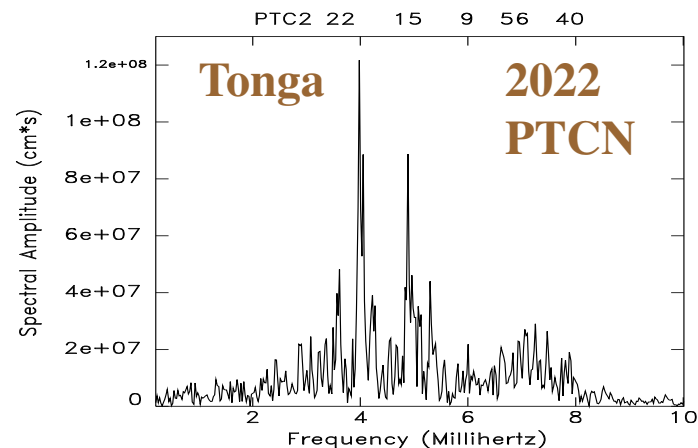
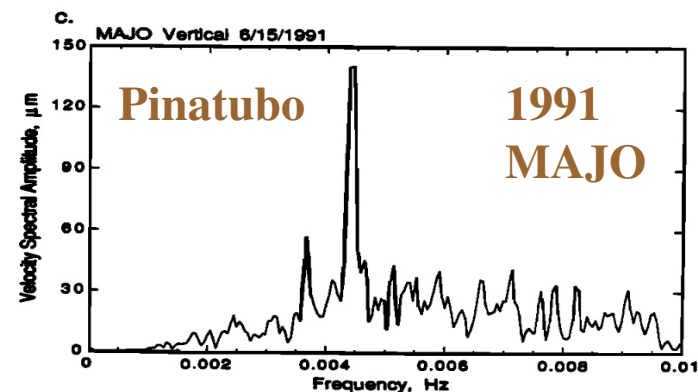
EXCEPT in the range 0.15–0.25 mHz (400–550 s)



→ In this study, we will focus on the lower-frequency part of the spectrum, where the air-wave ( $GR_0$ ) and tsunami modes are essentially uncoupled.

Additionally, at higher frequencies, selective resonance can take place where dispersion curves for  $S$  and  $GR$  modes intersect, also involving the spheroidal modes of the Solid Earth.

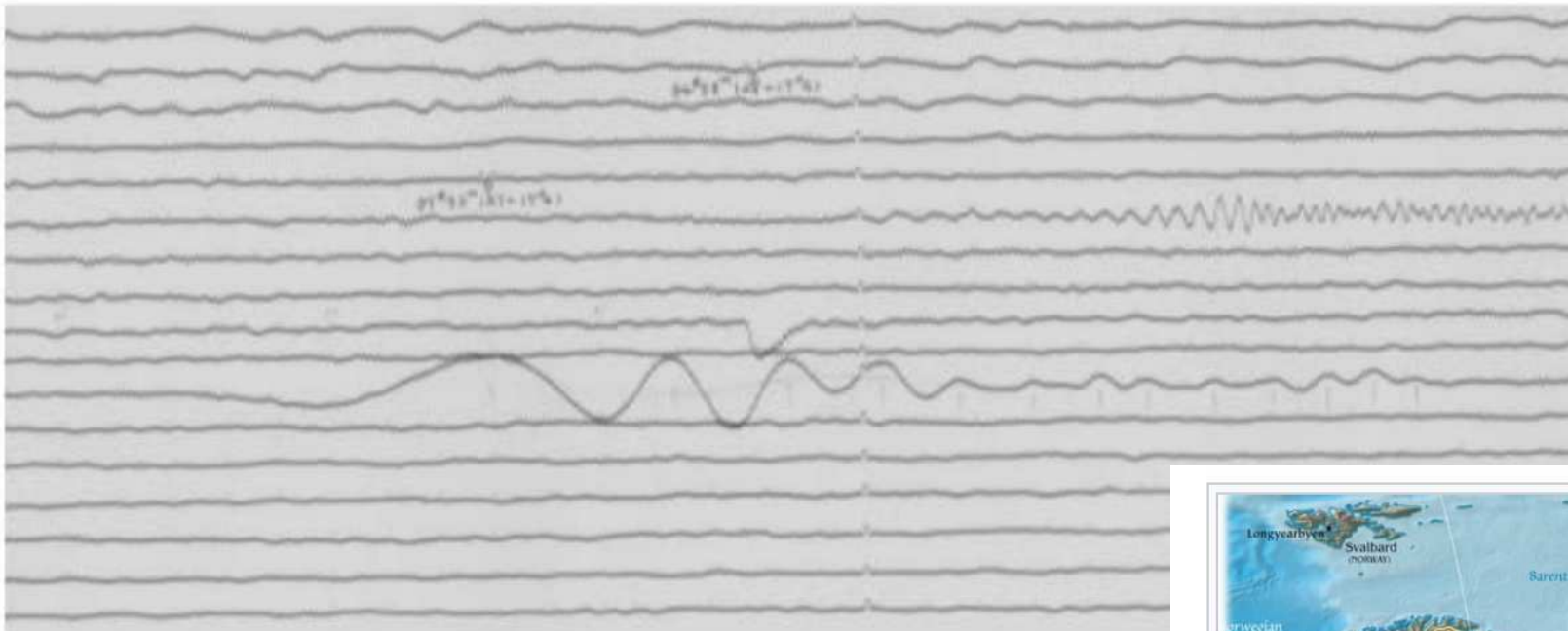
→ This can lead to largely monochromatic signals as observed at Pinatubo (1991) [Kanamori et al., 1994], and (remember) on the 2022 Pitcairn seismogram.



# ANOTHER SPECTACULAR CASE OF AIR WAVES

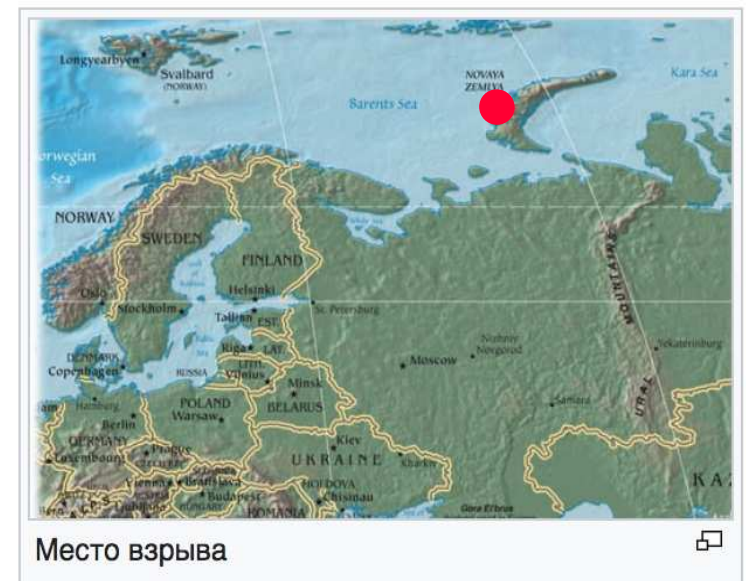
*"Tsar' Bomba", 30 OCT 1961*

*The largest ever nuclear test, 57 Mt*



*Air wave recorded on a seismometer  
at Tsukuba, Japan (6100 km)*

→ *No sea waves are known for this event...*



# TSUNAMI

*by*

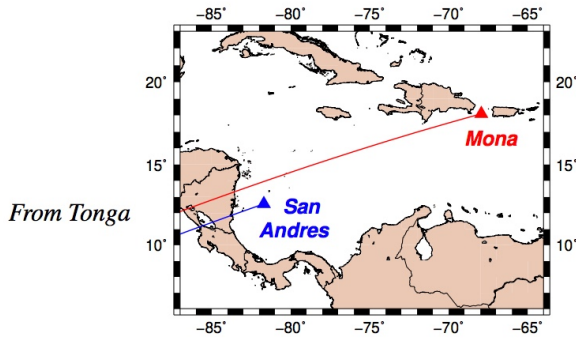
## NEXT-DAY AIR ?



Worldwide Services

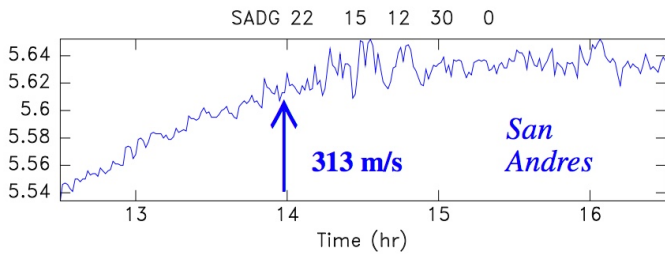
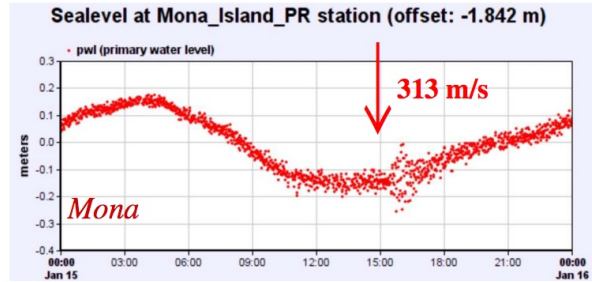
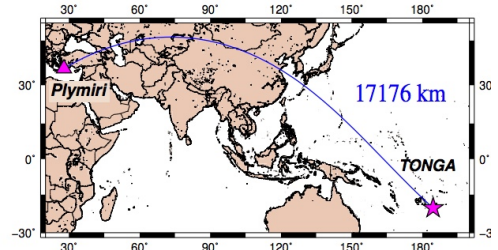
*Now... Even Across Continents !*

CARIBBEAN

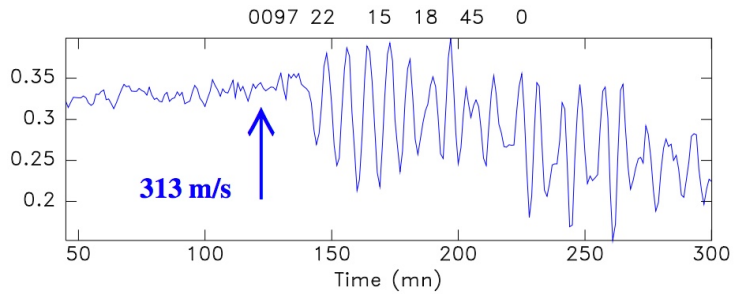
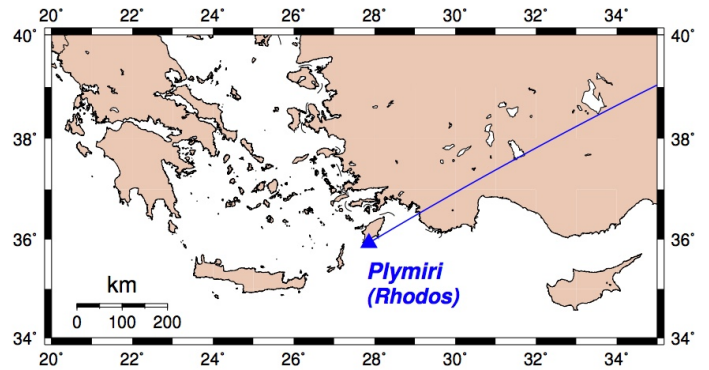


15 JAN 2022

The air-sea "tsunami" may appear to jump a land mass



MEDITERRANEAN (GREECE)



Maregram (m) at Plymiri, Rhodes (Greece), 15 JAN 2022

*Of course, after crossing a continent the real tsunami disappears and only the air wave is left, including its multiple passages*

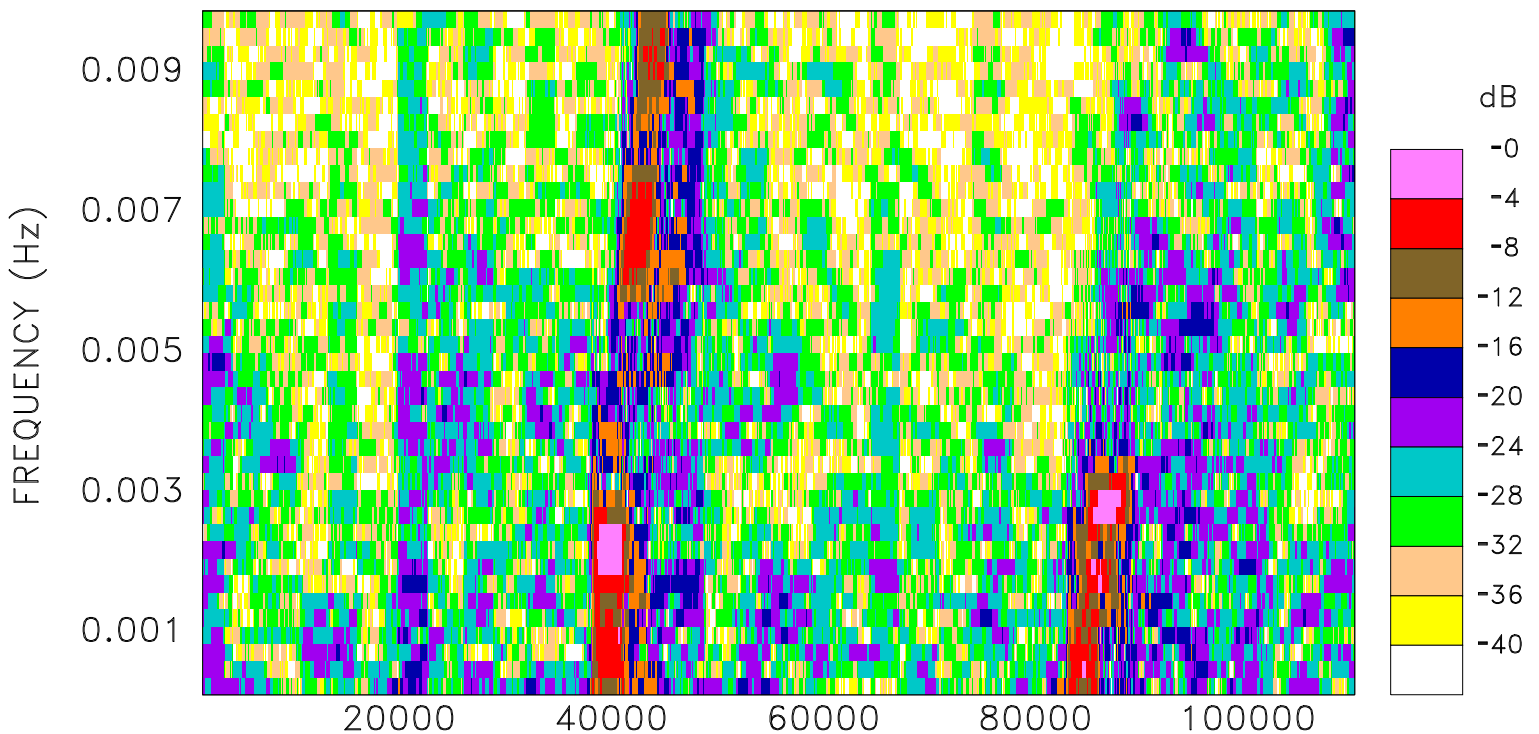
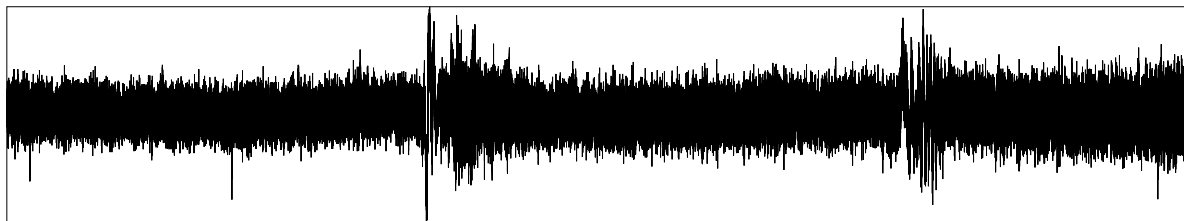
**GEOSCOPE STATION FDF**  
(Montagne Pelée, Martinique)  
**RAW E-W LHE RECORD**

**First  
passage  
 $A_1$**

**Second  
passage  
 $A_2$**

FDFE 22 15 5 0 0.0000

Peak-to-peak = 0.837E+04 du

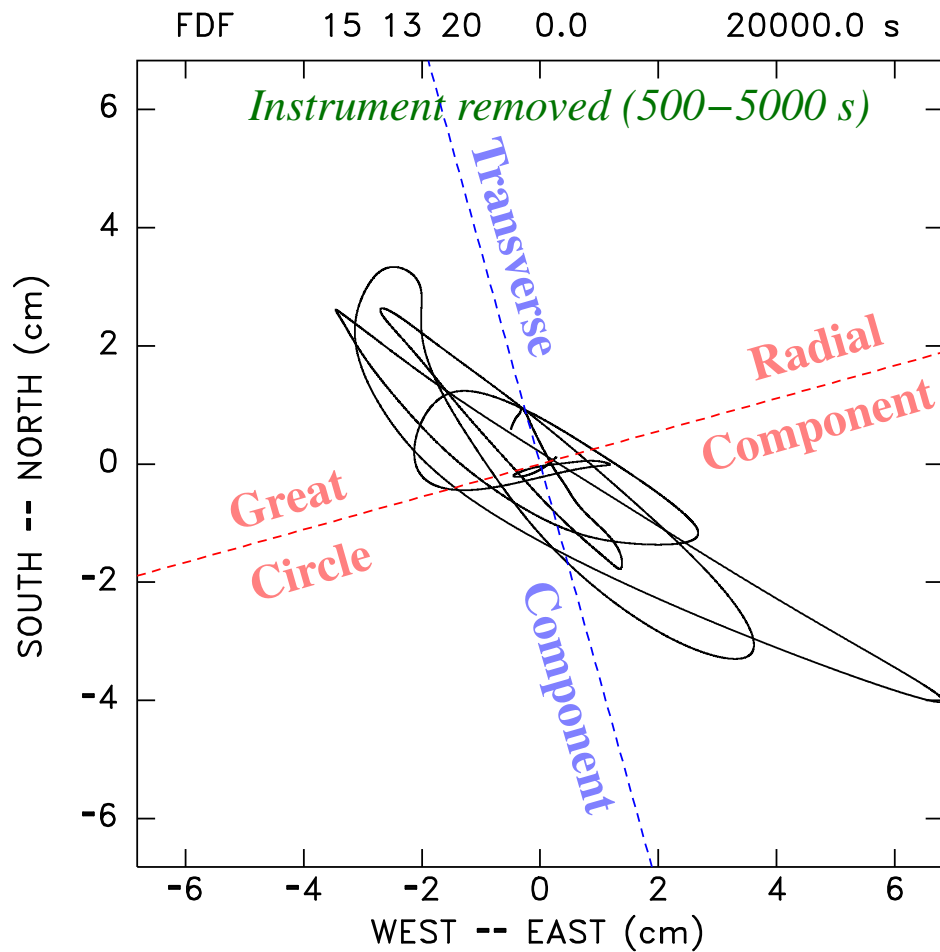


Window = 3000.0 s; Slice = 60. s.  
F-min = 0.00010 Hz; F-max = 0.01000 Hz.  
[ Plot bounds: 0.12207E-03 0.98877E-02 ]

Max. Spectral Amp. (0 dB) = 0.137E+07 du\*s  
Window used : 0.00 s; 111600.00 s.  
Plot window : 1470.00 s; 110070.00 s.



# HORIZONTAL PARTICLE MOTION at FDF (*Geoscope*)



## *First Passage of Air Wave*

Particle motion is  
polarized  $\sim 300^\circ$

about  $45^\circ$  from  
great circle ( $255^\circ$ ) !!

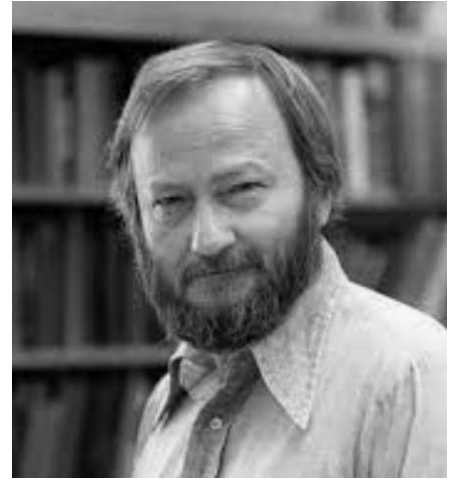
→ *Polarization (if any...) varies greatly, occasionally over VERY SHORT distances (<100 km)*

# THEORETICAL COMPUTATIONS of AIR-[SEA]-WAVES

*We use two codes*

## 1. " HASH "

**This code was written originally by D.G. Harkrider in the 1960s, following his theoretical work with F. Press.**



*It considers a flat-layered Solid Earth – [ Ocean ] – Atmosphere structure, and solves for the eigenfunction of the wave through an algorithm based on a Haskell propagator.*

## 2. " MODE "

**This code was written originally by H. Kanamori to compute the spheroidal (Rayleigh) modes of the Earth.**



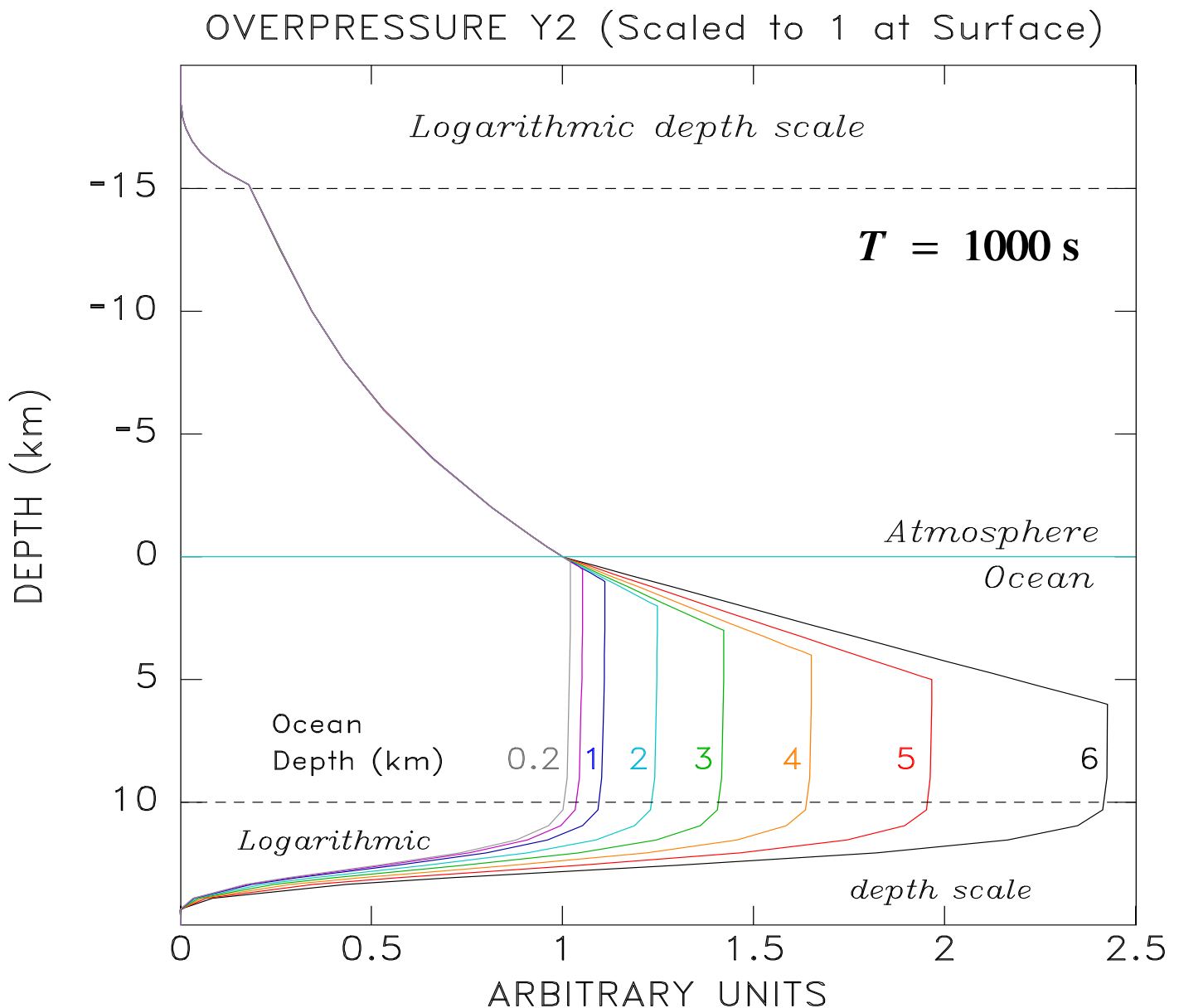
*It traces its ancestry to the works of C. Pekeris, H. Takeuchi and their students, notably M. Saito [1967].*

**It was adapted to the case of tsunamis by EAO following the work of S. Ward in the 1980s, and to the case of air waves by P. Lognonné, E. Clévéde and H. Kanamori in the 1990s.**

*In this study, we simply added about 40 atmospheric layers from DGH's model, and found that the program computes the air [-sea] waves without the need for any significant changes.*

# AIR WAVE EIGENFUNCTION IS LARGELY INDEPENDENT OF OCEAN DEPTH

When defined as the overpressure  $y_2$ , the structure of the air wave eigenfunction in the atmosphere is found to be largely independent of the presence (or depth) of the water column.

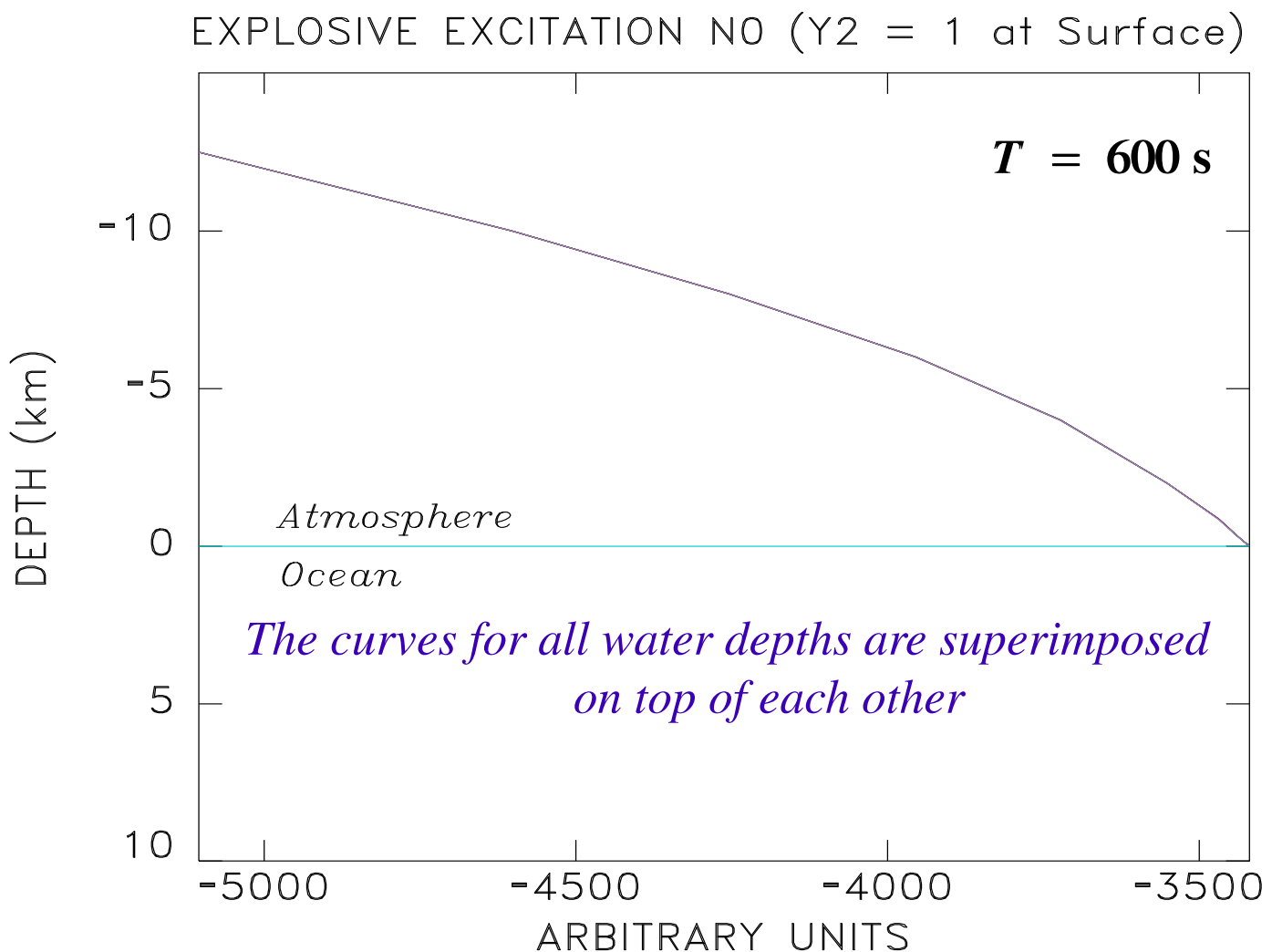


→ **This simply expresses that the coupling with the water remains very weak and does not appreciably affect the structure of the wave (nor its celerity).**

And then, because in a fluid layer, the normal mode excitation coefficient  $N_0$  for an explosive source

depends *ONLY ON THE COMPONENT*  $y_2$ ,

**$N_0$  is ALSO INDEPENDENT OF THE DEPTH OF THE OCEAN (or, actually of its presence)**



→ **The excitation of the air wave by an atmospheric explosion is not affected by the presence of an ocean below it.**

# WHAT is REALLY a $GR_0$ AIR WAVE ?

*Some back-of-the envelope thoughts based on simple Physics*

→ **The "tsunami" of the atmosphere ?**

*In a sense, yes, BUT .....*

→ *Such a tsunami should propagate (under the SWA) at a speed  $C = \sqrt{g H}$*

- But what is the height  $H$  of the atmosphere ? Probably, some average  $\langle H \rangle$  weighted by the particle density which decreases fast with height.
- Various attempts to obtain such an average height yield anywhere from 9 to 15 km.
- Note that this number also gives a reasonable estimate of the atmospheric pressure at the surface of the Earth, for an average density of about 2/3 the surface density.
- **This would fit very well a "tsunami" velocity of 313 m/s, requiring  $\langle H \rangle = 10$  km under the SWA.**

*BUT...*

**A close examination of the eigenfunction of  $GR_0$  shows that the energy is mostly... elastic. Strange ?**

- *We note that the speed of the "tsunami" (313 m/s) is VERY CLOSE to that of that of sound in the atmosphere (~ 340 m/s).*

*An interesting coincidence ?*

→ **Which brings in the question of the effect of finite compressibility on the structure and speed of a tsunami**

*This was investigated in the 2010s, notably by Watada et al., to explain early arrivals of tsunamis at teleseismic distances.*

*... but only for realistic values of the speed of sound in the ocean, which remains at least 7 times greater than that of a tsunami for all reasonable depths.*

→ **We investigate this effect in the case of a liquid 5–km deep ocean for compressional velocities ranging from 10 km/s (practically incompressible) to 0.1 km/s (less than the tsunami's SWA celerity).**

*We focus on the **elastic fraction of the potential energy** of the wave which should be close to 0 for a true "tsunami".*

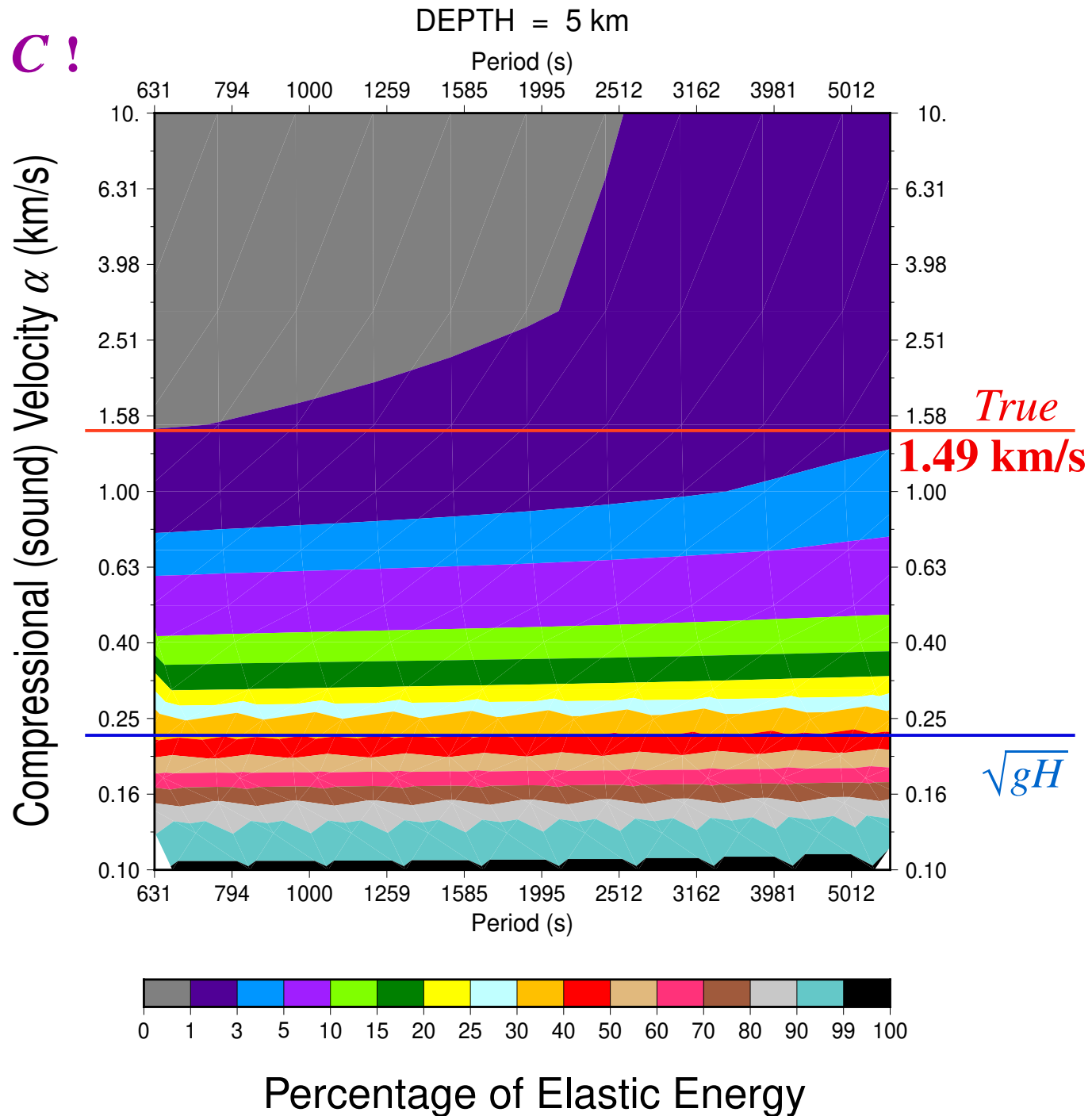
# SPEED of SOUND $\alpha$

## CLOSE to CELERITY $C$ !

### Effect of $\alpha$ on oceanic tsunami

*For realistic values of  $\alpha$ , the elastic energy is at most 2%, no influence on structure of tsunami wave.*

**For  $\alpha$  close to tsunami celerity, the energy becomes strongly elastic, the wave loses its structure as a tsunami.**



**313 m/s**

**340 m/s**

*An interesting coincidence ?*

*Maybe NOT.....*



## An interesting coincidence ?

*Maybe NOT.....*

- *SWA Tsunami velocity*

$$C = \sqrt{g \langle H \rangle} \qquad C^2 = g \langle H \rangle$$

- *Sound velocity*

$$\alpha = \sqrt{K_S / \rho}$$

→ For a **perfect gas**,  $K_S = \gamma P$  ( $\gamma = 1.4$  for  $N_2$ ,  $O_2$ ).

$$\frac{C^2}{\alpha^2} = \frac{g \langle H \rangle \rho}{\gamma P}$$

In the relevant parts of the atmosphere,  $P$  is expected to be on the order of  $\rho g \langle H \rangle$ , where  $\langle H \rangle$  is an average value of the atmosphere thickness.

**THUS, THE 2 VELOCITIES WILL BE COMPARABLE  
AND A PERFECT GAS CANNOT SUPPORT A TSUNAMI  
WHOSE ENERGY IS MOSTLY GRAVITATIONAL**

# MAJOR THEORETICAL QUESTIONS

1. *In the Air-Sea Wave, which are the parameters controlling the ratio of the air wave amplitude (essentially its pressure at the bottom of the atmosphere) to the amplitude of the sea-surface disturbance (as recorded by a maregraph) ?*
- **How will this "impedance" depend on the depth of the water layer (and of course, frequency) ?**

**NOTE** that it has often been taken as

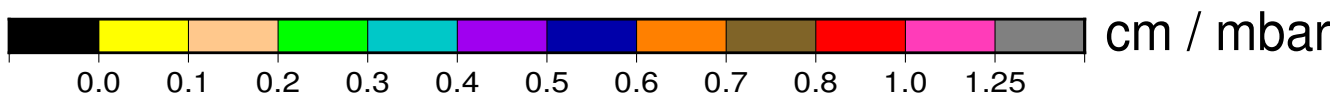
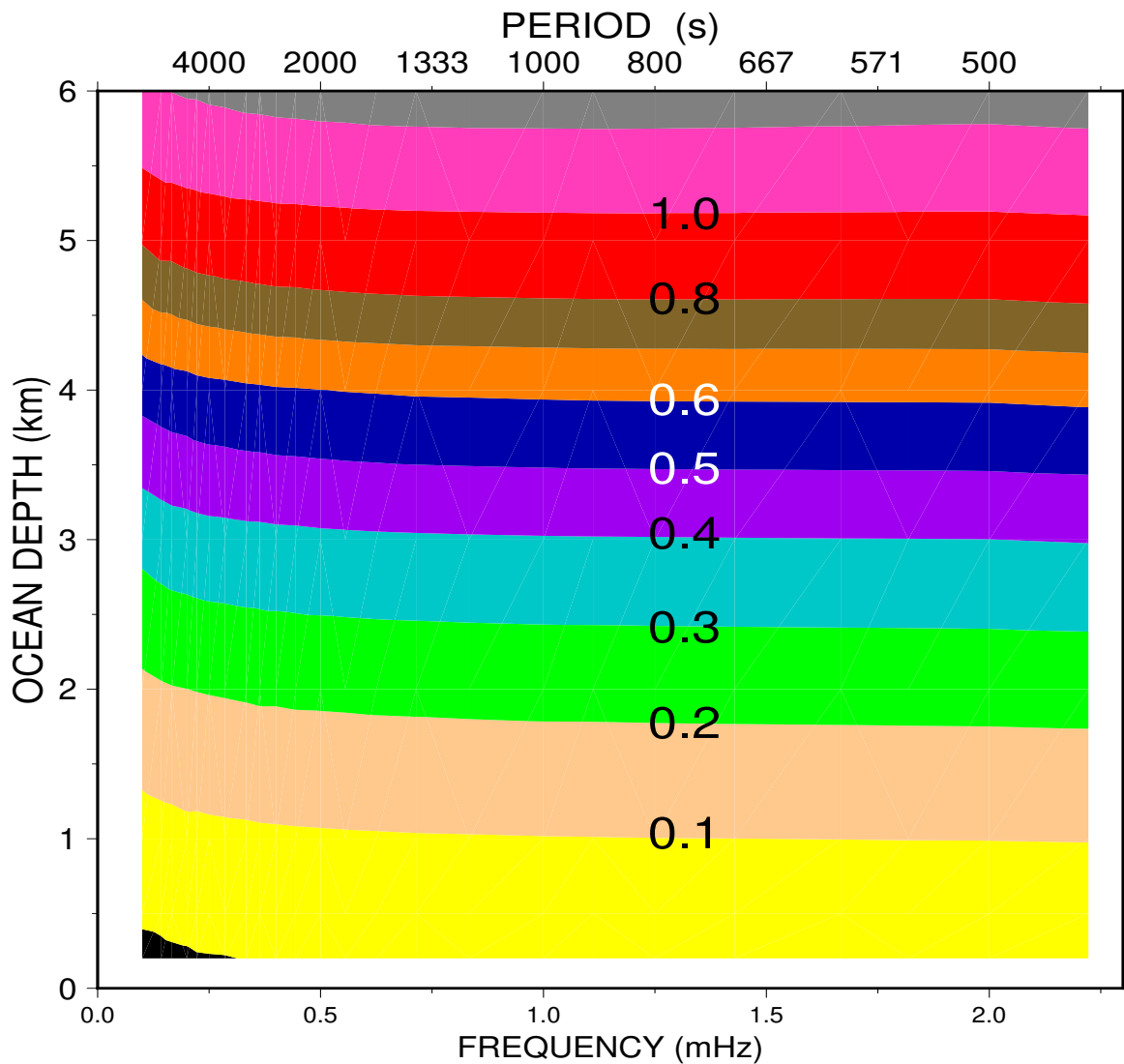
$$\rho_w g \approx 1 \text{ mbar/cm}$$

*i.e.*, the hydrostatic value, for example correlating the underpressure at the eye of a hurricane with the amplitude of the static storm surge

**WILL IT APPLY TO AN AIR-SEA WAVE ?**

# DYNAMIC RESPONSE RATIO

Results using D.G. Harkrider's 2-D code for flat-layered structures



indicate a weak dependence on frequency, but a much stronger one on water depth

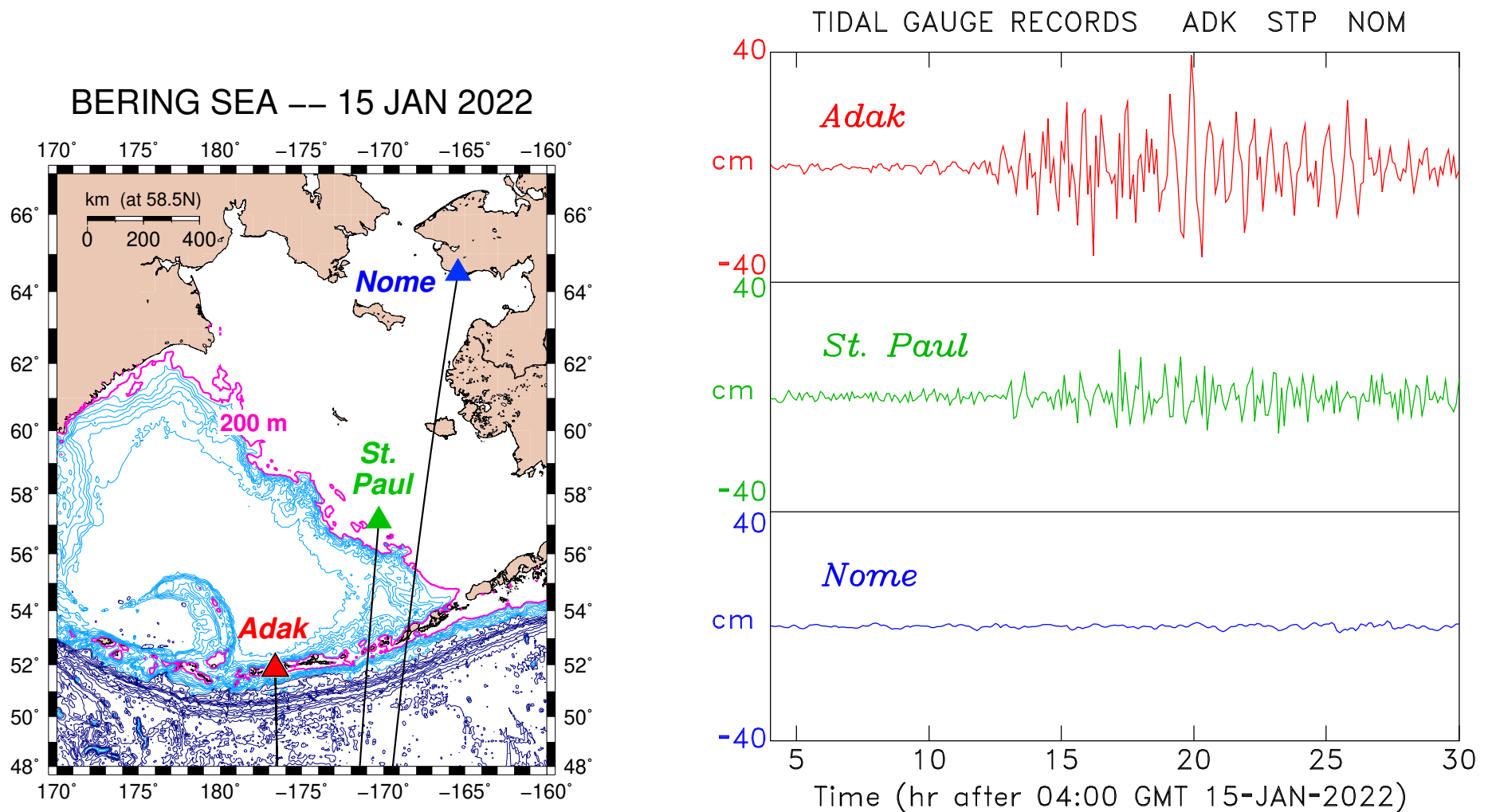
→ They generalize *Harkrider and Press*' [1967] results, who had considered only a depth of 5 km, thus approaching a dynamic ratio of 1 cm/mbar, which also corresponds to the hydrostatic value

$$\frac{1}{\rho_w g} = \frac{1}{1.03 * 981} \frac{\text{cm}}{\text{barye}} = 0.99 \frac{\text{cm}}{\text{mbar}}$$

but this coincidence is an artifact of their choice of depth.

*The decrease of the dynamic response with decreasing water depth predicts weaker coupling and smaller tsunami amplitudes in shallow basins, as exemplified in the Bering Sea during the Tonga explosion.*

**NOTE** wave disappearing at Nome, on shore of very shallow Northern Bering Sea (< 200 m).



# DIRECTIONS FOR FUTURE STUDY

- **Understand lateral heterogeneity in air wave records**

→ *Remember Professor Mohorovičić,*



*who switched careers from  
Meteorology to Seismology...*

A. Mohorovičić  
1857 – 1936

- **Relate Seismic Moment (about  $10^{25}$  dyn\*cm) to Energy (traditionally expressed in kt or Mt)**
- **Understand seismic recording of air wave by horizontal sensors**
- **Revisit properties of Tsar' Bomba Air wave**  
*(and search for any sea surface disturbance?)*